Non-Perturbative Effects & Heterotic String Vacua

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CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Overview

- Work with toroidal orbifold compactifications of the Heterotic string, e.g. $X_6 = T^6/\mathbb{Z}_N$
- Prove no-go theorem for dS minima for Kähler modulus + dilaton sector including non-perturbative effects in superpotential
- Attempt to extend no-go results to stronger non-perturbative effects in g_s & indicate potential loophole

No-Go Results

- Classical SUGRA [Maldacena,Nunez,'00]: no dS
- Subleading α' corrections [Green,Martinec,Quigley,Sethi,'11]: AdS, but no dS
- Infinite tower of α' [Gautason,Junghans,Zagermann,'12]: no AdS or dS
- All worldsheet effects [Kutasov,Maxfield,Melnikov,Sethi,'15]: no dS
- Gaugino Condensation [Quigley,'15]: no AdS or dS
- Gaugino Condensation, threshold effects, instantons*[Gonzalo, Ibanez,Uranga,'18]: AdS, numerically no dS

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What about non-perturbative effects of strength e^{-1/g_s}?

Two-Moduli Model

[Cvetic,Font,Ibanez,Lust,Quevedo,'91]
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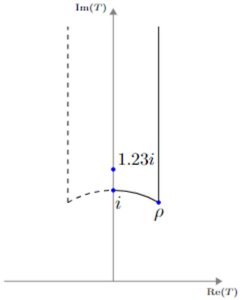
> Kähler Modulus T and Dilaton S
  - T has an $SL(2,\mathbb{Z})$ duality:

$$T \to \gamma \cdot T = \frac{aT+b}{cT+d}$$



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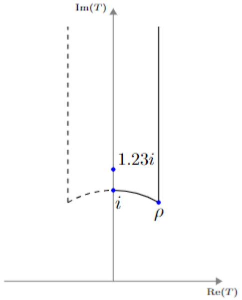

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- Superpotential

$$W(S,T) = \frac{\Omega(S)H(T)}{\eta^6(T)}$$

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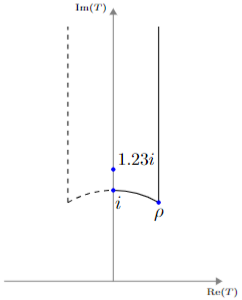

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$$\Omega(S) = h + \sum_a \Lambda_a^3 e^{-k_a S/b_a}$$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathcal{P}(j(T))$$

[Rademacher,Zuckerman,'38]
[Discontinuous Groups and Automorphic Functions - Lehner]

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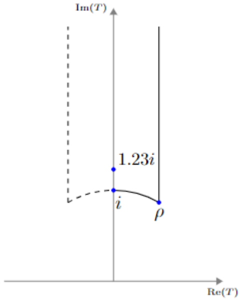

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$$A(S,\bar{S}) = \frac{K^{S\bar{S}}F_S\bar{F}_{\bar{S}}}{|W|^2} = \frac{K^{S\bar{S}}|\Omega_S + K_s\Omega|^2}{|\Omega|^2}$$

$$Z(T,\bar{T}) = \frac{1}{i(T-\bar{T})^3|\eta(T)|^{12}}$$

$$\hat{V}(T,\bar{T}) = \frac{-(T+\bar{T})^2}{3}\left|H'(T) - \frac{3i}{2\pi}H(T)\hat{G}_2(T,\bar{T})\right|^2$$

- Scalar Potential

$$V(S,T) = e^{\mathcal{K}}(K^{S\bar{S}}F_S\bar{F}_{\bar{S}} + K^{T\bar{T}}F_T\bar{F}_{\bar{T}} - 3\bar{W}W)$$
$$= e^{k(S,\bar{S})}Z(T,\bar{T})|\Omega(S)|^2\Big\{|H(T)|^2(A(S,\bar{S})-3) + \hat{V}(T,\bar{T})\Big\}$$

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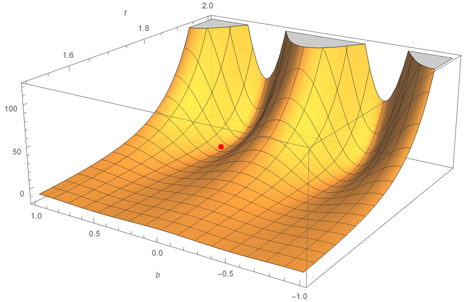

A dS No-Go Result

- Assume typical Heterotic dilaton stabilization:

$$\langle F_S \rangle = 0 \rightarrow A(S, \bar{S}) = 0$$

- Conditions for dS minimum at $T = T_0$:

$$V_0 = \Lambda^4 > 0$$

$$\partial_T V_0 = 0$$

$$\text{Hes}(V)_0 \ \& \ \partial_t^2 V_0 > 0$$

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Cannot both be positive

$\Updownarrow$

dS minima not possible

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$$V(S,\bar{S},\rho,\rho^*) = \frac{4^{4m+6}\pi^{12m+12}}{1225^m \times 27^{m+1}\Gamma^{18}(1/3)}|\Omega(S)|^2|\mathcal{P}(0)|^2 e^{k(S,\bar{S})}\left(A(S,\bar{S})-3\right)$$

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One method: Shenker-like Non-Perturbative Effects

Non-perturbative Contributions: Shenker-like Effects

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- Also in M-theory 1-loop amplitudes [Green & Rudra,'16]

$$S^{HO} \supset \frac{g_s^{-1/2}}{2^9(2\pi)^7 4!\ell_H^2} \int_{\mathcal{M}_{10}} \sqrt{-G}\, t_8 t_8 R^4 E_{3/2}(g_s^{-1})$$

$$E_{\frac{3}{2}}(g_s) = 2\zeta(3) g_s^{-3/2} + 2\zeta(2) g_s^{1/2} + \sum_{n\in\mathbb{Z}^+} 8\pi\sigma_{-1}(|n|) e^{-\frac{2\pi|n|}{g_s}} (1 + \mathcal{O}(g_s))$$

A Loophole Example

- Linear Multiplet Formalism: $L \supset \{\ell, \psi, B_2\}$

$$\langle \ell \rangle = \frac{g_s^2}{2}$$

$$\mathcal{L}_{KE} = \int d^4\theta E\Big(-2 + f(L)\Big) \qquad k(L) = \ln(L) + g(L)$$

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> Parametrize Shenker-like effects [Gaillard & Nelson,'07]+:

$$f(\ell) = \sum_{n=0} A_n \ell^{-q_n} e^{-B/\sqrt{\ell}} \qquad L\frac{df}{dL} = -L\frac{dg}{dL} + f$$

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- Scalar potential for single gaugino condensate:

$$V(\ell) = \frac{\mathcal{T}}{\ell}\Big[(1+\ell g')(1+b\ell)^2 - 3b^2\ell^2\Big] e^{g-(f+1)/b\ell}$$

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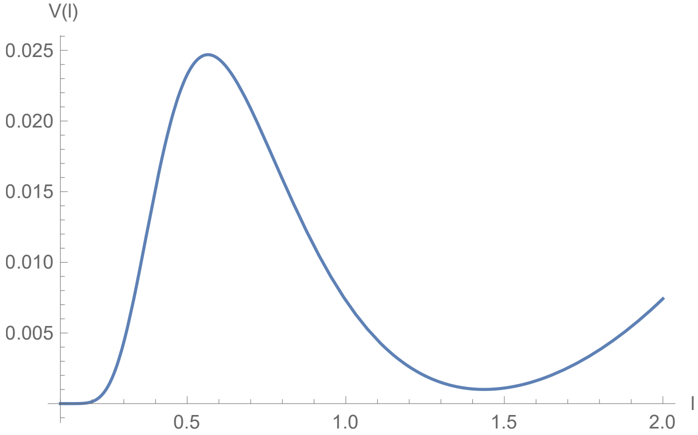

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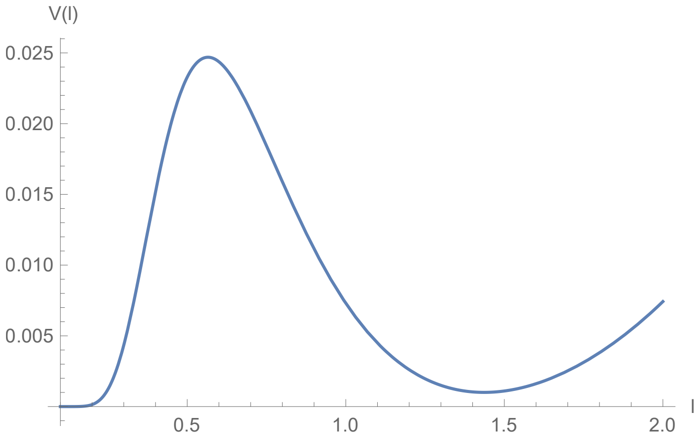

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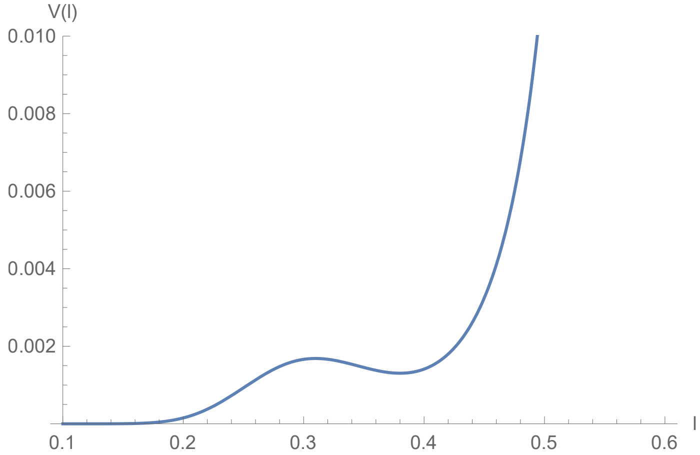

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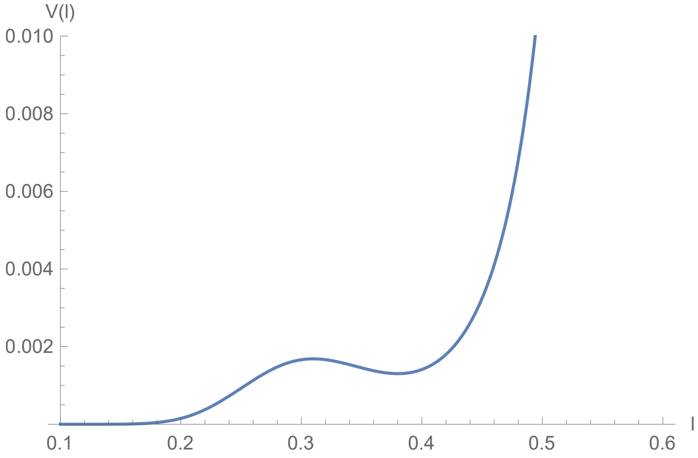

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$$\langle e^{-B/\sqrt{\ell}} \rangle \sim 3.7 \times 10^{-5}$$

$$\langle e^{-2B/\sqrt{\ell}} \rangle \sim 1.4 \times 10^{-9}$$

Conclusions

- Provided a no-go theorem $\Rightarrow$ extends connection between modular-invariance and swampland conjectures
- Illustrated a loophole in dS no-go arguments using Shenker-like e^{-1/g_s} effects in the Kähler potential
- Many potential future directions - most obvious is a better understanding of Shenker-like effects to close loophole or realize meta-stable dS in Heterotic string constructions
- Silverstein-like Effects - $\delta\mathcal{L} \sim e^{-1/g_s^{1/2}}$

Thank you!

Contact

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